oxidizer from the burning surface is then the integral of Eq. (6) over all possible ε_{ox} or

$$\dot{m}_{ox} = N \int_{o}^{\varepsilon_{ox}''} F_{ox}\varepsilon_{ox} \int_{\varepsilon_f'}^{\infty} m_{ox} F_f \, d\varepsilon_f \, d\varepsilon_{ox} \tag{7}$$

where ε_{ox}'' is the largest possible oxidizer surface. Since mixture ratio is preserved with means

$$\dot{m}_T = \dot{m}_{ox}/\alpha \tag{8}$$

The statistical formulation presented indicates that statistical combustion modeling is more complex than the analyses of Hermance and Beckstead, Derr, and Price suggest. However, the increase in complexity may be worthwhile because a basic assumption in one-dimensional combustion analyses of composite solid propellant combustion—substitution of a single flamelet for an ensemble—has been removed. In addition, virtually any one-dimensional combustion model may be employed to relate m_{ox} to its independent variables. Therefore, this formulation rather than degrading existing one-dimensional models provides a framework that may enhance their physical realism. Furthermore, there is a subtle but very powerful advantage to this statistical approach. With each flamelet "doing its own thing" in response to external stimuli and the response(s) to that stimuli determinable at the individual flamelet level through quasi-one-dimensional combustion models, flamelets favorable to and detrimental to a desired response can be identified. Consequently, desired responses can be enhanced by reducing the population of detrimental flamelets and augmenting the population of favorable flamelets. Therefore, this statistical formulation provides a systematic (and potentially quantitative) basis for the control of combustion phenomena through the manipulation of oxidizer particle sizes.

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Stability of Two-Hinged Circular Arches with Independent Loading Parameters

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Nomenclature

a = constant radius of the undeformed centroidal line of the arch rib
EI = bending stiffness
P = downward point load acting at the crown of an arch on the verge of buckling
P_o = P when $w = 0$
W = total weight of the arch rib
w = $W/2a\alpha$
w_o = critical own weight when $P = 0$
v_c = vertical displacement of the crown of the arch
2α = subtending angle of the arch

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WHEN nonshallow arches buckle at large deflections, their own deadweight is usually not a negligible quantity as compared to the applied load, and the problem of calculating critical loads becomes a two-parameter nonlinear boundary-value problem. Since it is very difficult to eliminate the effect of own weight in experimental work, it is of considerable importance to determine relationships between the two loads at the instant of impending instability of the arch.

For the foregoing reason, it was decided to calculate interaction curves for the critical values of the downward point load P at the crown of two-hinged circular arches and the uniformly distributed own weight w per unit length of the centroidal line. The exact nonlinear theory of the inextensible elastica was used in the calculations. This theory yields excellent results in the case of nonshallow slender arches.[1,2]

The governing equations of the theory of the initially curved elastica will not be derived herein, as they can be found in published literature.[3-5] A brief summary of the necessary equations is presented in Ref. 6. These equations were solved by means of an electronic digital computer to a high degree of accuracy. It was found that arches with subtending angles in the range $2\alpha = 60°–270°$ (see Fig. 1 of Ref. 6) buckle asymmetrically by sidesway, a bifurcation type of buckling.

The calculated critical values P and W of the interacting point load and the total weight of the arch, respectively, are made dimensionless by dividing them by the critical values P_o and W_o of these forces acting singly, i.e., $P = P_o$ when $W = 0$ and $W = W_o$ when $P = 0$. The P/P_o vs W/W_o interaction curves appear nearly straight (e.g., see Fig. 1) and can be approximated accurately by the simple equation

$$(P/P_o) + (W/W_o) = 1 \tag{1}$$

for all the values of the subtending angle 2α for which calculations have been carried out, namely, for $\alpha = 30°, 40°, 50°, 60°, 70°, 80°, 90°, 100°, 110°, 135°$. No consistency in regard to the convexity or concavity of the interaction curves is apparent; the curves for $\alpha = 30°, 40°, 50°, 135°$ are very slightly convex away

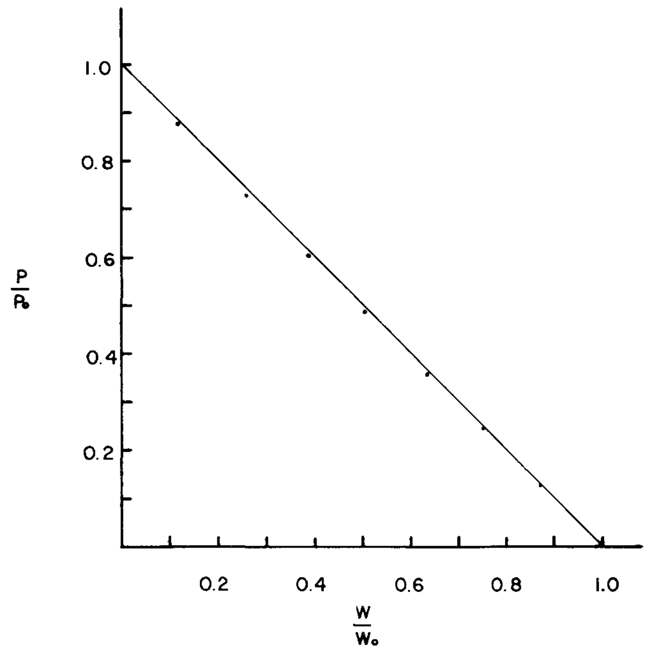

Fig. 1 Interaction curve for $\alpha = 80°$ (dots represent calculated points).

Table 1 Critical values

α	$w = 0$: $\frac{P_o a^2}{EI}$	$w = 0$: $\frac{v_c}{a}$	$P = 0$: $\frac{w_o a^3}{EI}$	$P = 0$: $\frac{W_o a^2}{EI}$	$P = 0$: $\frac{v_c}{a}$	$\frac{P_o}{W_o}$
30°	22.65	0.0327	35.72	37.41	0.00110	0.606
40°	17.15	0.0597	19.85	27.72	0.00350	0.619
50°	13.82	0.0953	12.42	21.67	0.00825	0.638
60°	11.44	0.1353	8.225	17.23	0.01614	0.664
70°	9.462	0.1711	5.574	13.62	0.0268	0.695
80°	7.617	0.1921	3.755	10.49	0.0385	0.726
90°	5.861	0.1946	2.498	7.846	0.0495	0.747
100°	4.287	0.1828	1.607	5.611	0.0573	0.764
110°	2.975	0.1607	0.999	3.835	0.0609	0.776
120°	1.944	0.1334	0.596	2.495	0.0601	0.779
135°	0.886	0.0882	0.248	1.169	0.0508	0.758

from the origin; the curve for $\alpha = 60°$ is in part convex and in part concave; and the remaining curves are slightly concave. Only one of the interaction curves ($\alpha = 80°$) is presented in this Note, Fig. 1, in order to illustrate the degree of curvature. The determination of the character of an interaction curve (stability boundary) is often a significant problem,[7] for, if the curve is convex away from the origin (away from the region of stability), the straight-line approximation, Eq. (1), yields critical load values which are on the safe side.

Equation (1) together with Table 1 will enable the designer rapidly and accurately to estimate the critical (buckling) loads for two-hinged circular arches. With these data we observe that the (modified) critical point load P is obtained by subtracting from P_o the values $(P_o W/W_o) =$ from $0.61W$ to $0.78W$ for arches with $\alpha = 30°$ to $120°$. Lind's suggested values[8] of $(P_o W/W_o) =$ from $0.50W$ to $0.67W$ are good for arches shallower than $\alpha = 60°$. It is interesting to note that the values of $(P_o W/W_o)$ increase with increasing α up to about $\alpha = 120°$ and then start decreasing.

Now let us make use of the calculated results to re-examine the experimental data in Ref. 9. The total weight W of a semicircular arch specimen of radius $a = 10$ in. was 0.255 lb or, since $EI/a^2 = 0.677$ lb, $W = 0.377EI/a^2$. Substitution of this value in Eq. (1), in which, according to Table 1, $P_o = 5.86EI/a^2$, yields $P = 5.58EI/a^2$ or 3.78 lb. The experimental plot of the point load vs the horizontal crown displacement[9] deviates from the load axis at $P = 3.7$ lb, a discrepancy of less than 3%.

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Time-Asymptotic Solution for Sphere-Cones in Hypersonic Flow

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Introduction

Currently, spherically-blunted cones are receiving prime attention as planetary entry configurations since such shapes have favorable ballistic coefficients, ability to tailor convective and radiative environments and efficient internal packaging characteristics. Several different approaches to solve these blunt body problems are currently available in computer program form. For small angled spherically-blunted cones, the inverse method combined with the method of characteristics has been successfully applied.[1] The method of integral relations[2] or its combination with the method of characteristics[3] has been used for either large or small cone angles. However, these methods cannot provide solutions to the blunt body problem over an intermediate range of cone angles where mixed supersonic-subsonic flow occurs on the conical portion of the body.

The purpose of this Note is to demonstrate the applicability of the time-asymptotic method to a full range of cone angles. This method can treat mixed subsonic and supersonic regions because of its technique of transforming the flow equations into hyperbolic form. Furthermore, it provides detailed flowfield properties including sonic line location and encounters no difficulty throughout the complete range of body geometry. The ensuing analysis is based on the computer solution of Barnwell[4] which is modified to treat the body surface and sonic corner more accurately.

Analysis

The fluid dynamic equations for an inviscid flow of a perfect gas in the shock layer can be cast in the divergence form using a body-oriented coordinate system as

$$(\partial F/\partial t) + (\partial P/\partial x) + (\partial Q/\partial y) + R = 0 \tag{1}$$

where

$$F = \lambda r \begin{vmatrix} \rho \\ \rho u \\ \rho v \\ \rho E \end{vmatrix}, \quad P = r \begin{vmatrix} \rho u \\ p + \rho u^2 \\ \rho uv \\ \rho uH \end{vmatrix}, \quad Q = \lambda r \begin{vmatrix} \rho v \\ \rho vu \\ p + \rho v^2 \\ \rho vH \end{vmatrix},$$

$$R = \begin{vmatrix} 0 \\ -p\lambda \cos\Theta + Kr\rho uv \\ -p\lambda \sin\Theta - Kr(p + \rho u^2) \\ 0 \end{vmatrix} \tag{2}$$

The boundary conditions behind the shock are specified by the Rankine-Hugoniot relations for a moving shock with its velocity determined by a characteristic compatability relation.

In order to obtain an accurate distribution of surface flow variables, the boundary conditions should reflect a true description of the flow at the surface. Therefore, instead of using the flow equations directly as adopted by Barnwell,[4] the surface flow properties are determined using the unsteady characteristic compatability relations which represent more accurately the physical aspects of the flow on the surface. Furthermore, the

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